QUADRATIC WORD PROBLEMS

Determining Maximum and Minimum Values

Example 1

A model rocket is launched from the roof of a building. Its flight path is modeled by $h = -5t^2 + 30t + 10$ where h is the height of the rocket above the ground in metres and t is the time after the launch in seconds.

What is the rocket's maximum height?

$h = -5(t^2 - 6t) + 10$

$= -5(t^2 - 6t + 9 - 9) + 10$

$= -5((t-3)^2 - 9) + 10$

$= -5(t-3)^2 + 45 + 10$

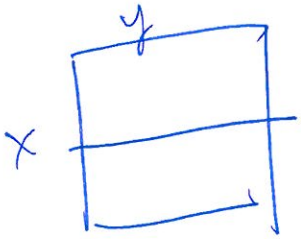
$= -5(t-3)^2 + 55$

$\therefore$ the max height is 55 m.

Example 2

A rectangular field will be fenced on all four sides. There will also be a line of fence across the field, parallel to the shorter side.

If 900 m of fencing are available, what dimensions of the field will produce the maximum area? What is the maximum area?



Let x rep. width

Let y rep. length.

$P = 900m$

$P = 2x + 3y$

$y = \frac{P - 2x}{3}$

$A = xy$

$A = x\left(\frac{900 - 2x}{3}\right)$

$= 300x - \frac{2}{3}x^2$

$= -\frac{2}{3}x^2 + 300x$

$= -\frac{2}{3}(x^2 - 450x)$

$= -\frac{2}{3}(x^2 - 450x + 50625 - 50625)$

$= -\frac{2}{3}((x - 225)^2 - 50625)$

$A = -\frac{2}{3}(x - 225)^2 + 33750$

$x = 225$, $y = \frac{900 - 2(225)}{3}$

$y = 150$

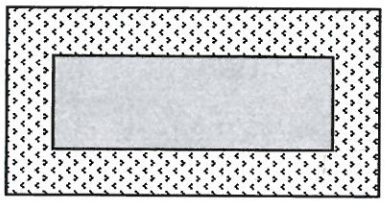
$\therefore$ the dimensions will be 225 x 150, with an area of 33750

QUADRATIC WORD PROBLEMS

Solving Quadratic Equations

Example 1

A rectangular lawn measuring 8 m by 4 m is surrounded by a flower bed of uniform width. The combined area of the lawn and the flower bed is 165 m^2. What is the width of the flower bed?



Let x rep the width of the flower bed

$A = l \times w$

$A = (8+2x)(4+2x)$

$165 = 32 + 16x + 8x + 4x^2$

$165 = 4x^2 + 24x + 32$

$0 = 4x^2 + 24x + 32 - 165$

$0 = 4x^2 + 24x - 133$

$0 = 4(x^2 + 6x) - 133$

$0 = 4(x^2 + 6x + 9 - 9) - 133$

$0 = 4((x+3)^2 - 9) - 133$

$0 = 4(x+3)^2 - 36 - 133$

$0 = 4(x+3)^2 - 169$

$169 = 4(x+3)^2$

$\frac{169}{4} = (x+3)^2$

$\pm\sqrt{\frac{169}{4}} = x+3$

$\pm\frac{13}{2} = x+3$

$\pm\frac{13}{2} - 3 = x$

$x = 3.5$ $x = -9.5$

∴ the width of the flower bed is 3.5 m.

Example 3

Tickets to a school dance cost $4 and the projected attendance is 300 people. For every $0.10 increase in ticket price, the dance committee projects that attendance will decrease by 5.

a) Determine the dance committee's greatest possible revenue.

Let x rep # of ticket price increases.

Let y rep total revenue.

$y = (4 + 0.1x)(300 - 5x)$

$y = 1200 - 20x + 30x - 0.5x^2$

$y = -0.5x^2 + 10x + 1200$

$y = -0.5(x^2 - 20x + 100 - 100) + 1200$

$y = -0.5(x - 10)^2 + 50 + 1200$

$y = -0.5(x - 10)^2 + 1250$

∴ the greatest revenue is $1250.

b) What ticket price will produce the greatest revenue?

$x = 10$

↳ x rep. # of ticket increases.

∴ $0.1(10) = \$1$ increase.

$1 + 4 = \$5$

∴ the ticket will cost $5.

QUADRATIC WORD PROBLEMS

Solving Quadratic Equations

Example 1

A water balloon is catapulted into the air so that its height h, in metres, after t seconds is $h = -4.9t^2 + 27t + 2.4$

a) How high is the balloon after 1 second?

$t = 1$

$h = 24.5$

b) For how long is the balloon more than 30 m high?

$30 = -4.9t^2 + 27t + 2.4$

$0 = -4.9t^2 + 27t - 27.6$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$t = \dfrac{-27 \pm \sqrt{27^2 - 4(-4.9)(-27.6)}}{2(-4.9)}$

$t = \dfrac{-27 \pm \sqrt{729 - 540.96}}{-9.8}$

$t = \dfrac{-27 \pm \sqrt{188.04}}{-9.8}$

$t = \dfrac{-27 \pm 13.71}{-9.8}$

$t = 1.36$ $\quad t = 4.154$

$4.15 - 1.36 = 2.79$

∴ the balloon is above 30 m for 2.79 sec

c) What is the maximum height of the balloon?

$h = -4.9(t^2 - 5.51t) + 2.4$

$= -4.9(t^2 - 5.51t + 7.59 - 7.59) + 2.4$

$= -4.9(t - 2.76)^2 + 37.19 + 2.4$

$= -4.9(t - 2.76)^2 + 39.59$

∴ the max height is 39.59 m.

d) When will the balloon burst as it hits the ground?

$h = 0$.

$0 = -4.9t^2 - 27t - 2.4$

$t = \dfrac{-27 \pm \sqrt{27^2 - 4(-4.9)(-2.4)}}{2(-4.9)}$

$t = \dfrac{-27 \pm \sqrt{729 - 47.04}}{-9.8}$

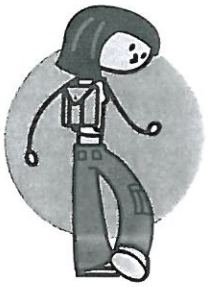
$t = \dfrac{-27 \pm \sqrt{681}}{-9.8}$

$t = \dfrac{-27 \pm 26.11}{-9.8}$

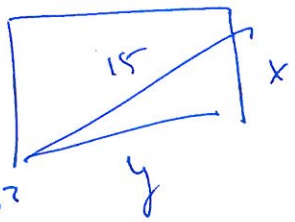
$t = 0.09$ $\quad t = 5.42$

∴ the balloon hits the ground at 0 sec & 5.42 sec.

Example 2



Nancy walks 15 m diagonally across a rectangular field. She then returns to her starting position along the outside of the field. The total distance she walks is 36 m. What are the dimensions of the field?



Let x rep width of the field
Let y rep length of the field.

$c^2 = a^2 + b^2$

$15^2 = x^2 + y^2$

② $225 = x^2 + y^2$

$15 + x + y = 36$

$x + y = 21$

① $y = 21 - x$

① → ②

$225 = x^2 + (21-x)^2$

$225 = x^2 + x^2 - 42x + 441$

$0 = 2x^2 - 42x + 441 - 225$

$0 = 2x^2 + 42x + 216$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{42 \pm \sqrt{(42)^2 - 4(2)(216)}}{2(2)}$

$x = \dfrac{42 \pm \sqrt{1764 - 1728}}{4}$

$x = \dfrac{42 \pm \sqrt{36}}{4}$

$x = \dfrac{42 \pm 6}{4}$

$x = 12 \qquad x = 9$

$y = 21 - x$

If $x = 12$, $y = 9$

If $x = 9$, $y = 12$

∴ the dimensions of the field are 9 x 12.