The set-up for the applications in this chapter will be rational equations. The keyword, ***reciprocal***, appears often. Remember that the reciprocal of a number is 1 divided by that number.

A. Number Problems (*w/ keyword reciprocal*)

One positive integer is 5 more than the other. When the reciprocal of the larger number is subtracted from the reciprocal of the smaller the result is $\frac{5}{14}$. Find the two integers.

Let x = the smaller positive integer

then $x+5$ = the larger integer.

Set-up $\rightarrow$ $\frac{1}{x} - \frac{1}{x+5} = \frac{5}{14}$

$$\frac{1}{x}\cdot x(x+5)\cdot 14 - \frac{1}{x+5}\cdot x(x+5)\cdot 14 = \frac{5}{14}\cdot x(x+5)\cdot 14$$

$$14(x+5) - 14x = 5x(x+5)$$

$$14x + 70 - 14x = 5x^2 + 25x$$

$$70 = 5x^2 + 25x$$

$$0 = 5x^2 + 25x - 70$$

$$0 = (x+7)(5x-10)$$

$x+7=0$ or $5x-10=0$

$x=-7$ negative

$5x=10$

$x=2$

The two integers are 2 and 7.

The difference between two integers is 5. If the reciprocal of the smaller is added to twice the reciprocal of the larger the result is $\frac{23}{66}$. Find the two integers.

Let x = the larger integer

$x-5$ = the smaller integer

$\left(\begin{aligned} x-y&=5 \\ -y&=-x+5 \\ y&=x-5 \end{aligned} \right)$

Set-up $\rightarrow$ $2\left(\frac{1}{x}\right) + \frac{1}{x-5} = \frac{23}{66}$

$$\frac{2}{x}\cdot 66x(x-5) + \frac{1}{x-5}\cdot 66x(x-5) = \frac{23}{66}\cdot 66x(x-5)$$

$$132(x-5) + 66x = 23x(x-5)$$

$$132x - 660 + 66x = 23x^2 - 115x$$

$$198x - 660 = 23x^2 - 115x$$

$$0 = 23x^2 - 313x + 660$$

$$0 = (23x-60)(x-11)$$

$23x-60=0$ or $x-11=0$

$23x=60$

$x=60/23$

$x=11$

The two integers are 11 and 6.

The difference between the reciprocals of two consecutive positive odd integers is $\frac{2}{15}$. Find the integers.

Let x = the first positive odd integer.
$x+2$ = the next odd integer

Set-up $\rightarrow$ $\frac{1}{x} - \frac{1}{x+2} = \frac{2}{15}$

$$\frac{1}{x}15x(x+2) - \frac{1}{x+2}15x(x+2) = \frac{2}{15}\cdot 15x(x+2)$$

$$15(x+2) - 15x = 2x(x+2)$$
$$15x + 30 - 15x = 2x^2 + 4x$$
$$30 = 2x^2 + 4x$$
$$0 = 2x^2 + 4x - 30$$

$$0 = 2(x^2 + 2x - 15)$$
$$0 = 2(x-3)(x+5)$$
$$x - 3 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 3 \qquad x = -5$$
negative

The two integers are 3 and 5.

We have two equivalent formulas to choose from when solving ***work-rate problems***. If two people are working together on a job then their rates add and they can perform the job working together in a shorter amount of time.

If we let x = time it takes person 1 to complete the task then his work rate is $\frac{1}{x}$. In other words, he can complete the 1 job in x number of hours. If we let y = time it takes person 2 to complete the task and t = time it takes with both working together we get the following formulas:

Work – Rate Formulas: $\frac{1}{x} + \frac{1}{y} = \frac{1}{t}$ **or alternatively** $\frac{t}{x} + \frac{t}{y} = 1$

B. Work-Rate Problems

Bill's garden hose can fill the pool in 12 hours. His neighbor has a hose that can fill the pool in 15 hours. How long will it take to fill the pool using both hoses?

Let t = time it takes to fill the pool with both hoses.

$$\underbrace{\text{Rate of Bill's hose} + \text{Rate of Neighbor's hose} = \text{Rate of Both hoses}}$$

$$\frac{1}{12} + \frac{1}{15} = \frac{1}{t}$$

The rates add, not the times.

$$\frac{1}{12}60t + \frac{1}{15}60t = \frac{1}{t}60t$$
$$5t + 4t = 60$$
$$9t = 60$$

$$t = \frac{60}{9}$$
$$= 6\tfrac{6}{9}$$
$$= 6\tfrac{2}{3}$$

Both hoses can fill the pool in $6\frac{2}{3}$ hours.

Joe can complete his yard work in 3 hours. If is son helps it will only take 2 hours working together. How long would the yard work take if is son was working alone?

Let y = time it takes Joe's son to do the yard.

Set-up → $\frac{1}{3} + \frac{1}{y} = \frac{1}{2}$

$$\frac{1}{3}\cdot 6y + \frac{1}{y}\cdot 6y = \frac{1}{2}\cdot 6y$$

$$2y + 6 = 3y$$

$$6 = y$$

Working alone, it would take Joe's son 6 hours.

Norm and Cliff can paint the office in 5 hours working together. Being a professional painter, Norm can paint twice as fast as Cliff. How long would it take Cliff to paint the office by himself?

Let x = time it takes Norm to paint the office alone.

$2x$ = time it will take Cliff working alone

Cliff is slower so it will take him twice as long as Norm working alone.

Set-up → $\frac{1}{x} + \frac{1}{2x} = \frac{1}{5}$

$$\frac{1}{x}\cdot 10x + \frac{1}{2x}\cdot 10x = \frac{1}{5}\cdot 10x$$

$$10 + 5 = 2x$$

$$15 = 2x$$

$$\frac{15}{2} = x$$

Back substitute:

$2x = 2\left(\frac{15}{2}\right) = 15$

So it would take Cliff 15 hours to paint the office alone.

We have set up ***uniform motion problems*** using the formula $D = r\,t$. For the following motion problems we will need the equivalent formula $\frac{D}{r} = t$ to set up the equations.

C. Uniform Motion Problems

The first leg of Mary's road trip consisted of 120 miles of traffic. When the traffic cleared she was able to drive twice as fast for 300 miles. If the total trip took 9 hours how long was she stuck in traffic?

Let x = Mary's speed in traffic

$2x$ = speed after traffic

	D =	r ·	t
1^{st} leg	120	x	$\frac{120}{x}$
2^{nd} leg	300	$2x$	$\frac{300}{2x}$
			9 hrs ← Total

$\frac{D}{r} = \frac{r\cdot t}{r}$

$\frac{D}{r} = t$

Set-up

$$\frac{120}{x} + \frac{300}{2x} = 9$$

$$\frac{120}{x}\cdot 2x + \frac{300}{2x}\cdot 2x = 9\cdot 2x$$

$$240 + 300 = 18x$$

$$540 = 18x$$

$$\frac{540}{18} = \frac{18x}{18}$$

$$30 = x$$

So $60 = 2x$

time in traffic was $\frac{120}{x} = \frac{120}{30}$

$= 4$ hrs.

She was in traffic for 4 hours.

A passenger train can travel 20mph faster than a freight train. If the passenger train can cover 390 miles in the same time it takes the freight train to cover 270 miles, how fast is each train?

let x = speed of freight train.
$x+20$ = speed of passenger train.

	D	= r	· t
freight train	270	x	$\frac{270}{x}$
Passenger train	390	$x+20$	$\frac{390}{x+20}$

"same time"

set-up

set-up → $\frac{270}{x} = \frac{390}{x+20}$

$270(x+20) = 390x$

$270x + 5400 = 390x$

$5400 = 120x$

$\frac{5400}{120} = \frac{120x}{120}$

$45 = x$

so $65 = x+20$

Freight train speed is 45mph and passenger train's speed is 65mph

Billy rode his skateboard 24 miles to his grandmother's house for the day. It was a rough ride so he borrowed his grandmother's bicycle for the return trip. Going twice as fast on the bicycle the return trip took 2 hours less time. What was his average speed on the bicycle?

let x = speed on skateboard
$2x$ = speed on Bike

	D	= r	· t
skateboard	24	x	$\frac{24}{x}$
Bicycle	24	$2x$	$\frac{24}{2x}$

set-up $\frac{24}{x} - \frac{24}{2x} = 2$

$\frac{24}{x} \cdot x - \frac{12}{x} \cdot x = 2 \cdot x$

$24 - 12 = 2x$

$12 = 2x$

$6 = x$

Billy averaged 6mph on the skateboard and 12mph on the bicycle.

Brett lives on the river 45 miles upstream from town. When the current is 2mph he can row his boat downstream to town for supplies and back in 14 hours. What is his average rowing speed in still water?

let x = Brett's rowing speed

	D	= r	· t
Downstream	45	$x+2$	$\frac{45}{x+2}$
Upstream	45	$x-2$	$\frac{45}{x-2}$
			14

Total time

$D = rt$

$\frac{D}{r} = t$

Set-up

$\frac{45}{x+2} + \frac{45}{x-2} = 14$

$\frac{45}{(x+2)}(x+2)(x-2) + \frac{45}{(x-2)}(x+2)(x-2) = 14(x+2)(x-2)$

$45(x-2) + 45(x+2) = 14(x+2)(x-2)$

$45x - 90 + 45x + 90 = 14(x^2-4)$

$90x = 14x^2 - 56$

$0 = 14x^2 - 90x - 56$

$0 = 2(7x^2 - 45x - 28)$

$0 = 2(7x+4)(x-7)$

$7x+4=0$ or $x-7=0$

$7x = -4$

$x = -4/7$ negative

$x = 7$

Brett's rowing speed in still water is 7mph.

Variation problems often set up as rational equations. Given two quantities *x* and *y* the following keywords indicate a particular relationship where *k* is called the variation constant.

Direct Variation:	**Inverse Variation:**	**Direct Variation:**
y ***varies directly as*** x or y *is* ***directly proportional*** *to* x $y = kx$	y ***varies inversely as*** x or y *is* ***inversely proportional*** *to* x $y = \frac{k}{x}$	y ***varies jointly*** *as* x or y *is* ***jointly proportional*** *to* x $y = kxy$

D. Variation Problems

Weight on Earth varies directly with the weight on the Moon. With his equipment, an astronaut weighs 360 pounds on earth but only 60 pounds on the moon. If another astronaut had landed on the moon that weighed 54 pounds with her equipment, how much would she weigh on Earth with equipment?

Let y = weight on Earth

x = weight on moon

Set-up → $y = kx$

To find k: $360 = k(60)$

$\frac{360}{60} = \frac{k(\cancel{60})}{\cancel{60}}$

$6 = k$

$\boxed{\text{model: } y = 6x}$

Now $y = 6(54)$

$= 324$

On earth she would weigh 324 lbs.

The weight of a body varies inversely as the square of its distance from the center of the Earth. If a person weighs 175 pounds on the surface of the earth ($r \approx 4000$ miles) how much will he weigh at 1000 miles above the Earths surface?

Let y = weight of the body

x = Distance from the center of the earth

set-up → $y = \frac{k}{x^2}$

To find k: $175 = \frac{k}{(4000)^2}$

$175(4000)^2 = K$

$2.8 \times 10^9 = K$

model: $y = \frac{2.8 \times 10^9}{x^2}$

Find y when $x = 4000 + 1000$

$y = \frac{2.8 \times 10^9}{(5000)^2} = \frac{2.8 \times 10^9}{2.5 \times 10^7}$

$= 1.12 \times 10^2$

$= 112$

He will weigh 112 lbs at 1000 miles up.